Grid Metric Gravity as an Alternative to General Relativity

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A new theory of gravity is derived which duplicates, up to the order of the square of m/r, all of the results of General Relativity that have been conclusively tested by observation. Called *Grid Metric Gravity*, this theory is linear, and unlike General Relativity, assumes as its fundamental physical quantity the classical gravitational potential. The particle Lagrangian depends on the line element ds in the same way as in General Relativity. Thus, Grid Metric Gravity is a true metric theory and automatically satisfies the Equivalence Principle. The theory is complete in that it does not require a thermodynamic Equation of State to determine the curvature of space-time. Grid Metric Gravity may point to an explanation of how matter creates Minkowski space, and suggests a Machian view of the universe.

I. INTRODUCTION

General Relativity has been considered our most accurate theory of gravity for almost a century. Yet in all that time, a full understanding of the theory has not been achieved. Today many physicists question General Relativity (GR) for a variety of reasons. Prominent among them is the fact that all attempts to quantize GR have so far had limited success [1-3]. GR also does not account for anomalies in the galactic rotation curve, wherein the outer bodies of galaxies orbit with a speed approaching a constant at large r in accordance with the Tully-Fisher law [4,5]. Moreover, GR does not predict the apparent observed acceleration of universal expansion except with the reintroduction of Einstein's cosmological constant **Λ**, which must be fine tuned in a way that is not fully explained [6-9]. As a result of these and other discrepancies, perhaps hundreds of theories of gravity have been proposed in recent decades [10-13]. Many of them are variants on GR involving changes to Einstein's Field Equations [14-16], or the introduction of scalar [17-19] or vector potentials [20]. For reviews of these theories see references [21-26].

There are other reasons GR might be questioned. That Einstein's Field Equations are incomplete without an independently postulated Equation of State is less than satisfying. Indeed, in many applications, the Equation of State (EoS) carries more information than the field equations themselves. Yet in actual practice, the EoS is often largely a product of conjecture, or tailored to give the desired result [27,28]. A complete theory of gravity independent of thermodynamics might be preferable from an aesthetic standpoint. Note also that most practical applications of GR utilize the Schwarzschild metric, a solution to Einstein's Field Equations (EFE) for the vacuum. Yet the Schwarzschild solution does not explicitly require an EoS. This points to an inconsistency in the underlying principles of the theory.

A few researchers might also object to the predicted existence of black holes, invisible bodies in which all matter is confined within an event horizon at r = 2m, a so-called coordinate singularity. This prediction distinguishes GR from both Newtonian gravity and classical electromagnetism, for which singularities exist only at r = 0.

GR presents another problem, usually considered a mere inconvenience, but which may be interpreted as an actual contradiction. The metric $g_{\mu\nu}$, which is determined by solving EFE, is nonlinear. Thus, for example, there is no way to add two Schwarzschild metrics to obtain the space-time curvature for two masses. Yet mass is presumed to add linearly. Why might this be a contradiction? Consider two masses m_1 and m_2. Each has a metric such that the time component is of the form $g_{00} = 1 - 2m/r$. If the masses reside together at r = 0, the metric becomes $g_{00} = 1 - 2(m_1 + m_2)/r$. Yet there is no simple mathematical operation on the two original metrics that gives the final result. This suggests that it is the masses that are fundamental physical quantities, not the metrics. Yet, according to EFE, the metric is the fundamental physical quantity.

Another feature of GR that is less than satisfactory relates to Einstein's Field Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci tensor, *R* the scalar curvature (not to be confused with the radius of the visible universe, also denoted *R*), $\kappa = -8\pi G/c^2$, and $T_{\mu\nu}$ is the stress-energy tensor. These equations have been subject to theory-independent observational tests only in the case of the Schwarzschild metric, a vacuum solution. Note however that EFE simplify significantly in the vacuum, for which $T_{\mu\nu} = R = 0$, becoming just $R_{\mu\nu} = 0$. *Have observational tests been conducted for the full EFE using a non-zero density distribution* $\rho(r)$ *?* Even if they have, the results would depend on the selected Equation of State, which is not known *a priori*, but is theory-dependent. Hence any such tests would not be conclusive. The Schwarzschild metric is thus the only feature of GR that has been verified observationally in a conclusive theory-independent way.

From this thinking there arises the conjecture that a complete theory of gravity can be constructed from Schwarzschild metrics alone. This idea is not new. A similar concept was proposed by Richard Lindquist and John Archibald Wheeler in 1957, and was called the *black hole lattice* theory. It is discussed in more detail in references [29,30], and is currently of interest in solving the cosmological *back-reaction* problem, a difficulty in the standard model arising from inhomogeneities in the mass density of the universe [31]. Here I refer to it as the *Schwarzschild Grid Model*, or SGM. The problem with SGM is again that the metrics cannot be directly combined, but must be "patched" together in a way not entirely rigorous according to the GR formalism. This implies that $g_{\mu\nu}$ is not the fundamental physical quantity.

Below, I introduce a theory of gravity inspired by the SGM concept, but for which the fundamental physical quantity is the classical gravitational potential. The metric $g_{\mu\nu}$ is regarded as a secondary quantity derived from the relevant potentials, and serves mainly to determine the particle Lagrangian. For a single mass, the formalism gives rise to a line element identical to the Schwarzschild line element up to order m^2/r^2. Thus it is approximately consistent with all conclusive observational tests of GR. The theory introduced here utilizes the cosmic coincidence $GM_U/c^2 R \sim 1$, where M_U is the presumed mass of the universe and *R* the so-called radius of the visible universe, or *co-moving horizon* [32]. The new theory is linear in that the fundamental physical quantities $\phi_i = -m_i/r_i$ add linearly. The line element depends on the metric in the usual way:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu .$$

which can be written in spherical coordinates as:

$$ds^2 = g_{00} dt^2 + g_{11} dr^2 - r^2 d\Omega^2$$

In the theory developed here, the Lagrangian for a test particle of mass m' is $L = m' ds/dt$, and depends on the metric just as in GR. Since all motion, as well as physically measurable observables such as time dilation and redshift [33], are determined by the metric, the theory qualifies as a full metric theory of gravity. Hence the Equivalence Principle is automatically satisfied. This theory will be called *Grid Metric Gravity*, or GMG.

II. GRID METRIC GRAVITY: THE FORMALISM

To develop the formalism of GMG, consider first for heuristic purposes an attempt to formulate a theory of gravity based on an irregular grid of Schwarzschild line elements

$$ds^2 = (1 - \frac{2m}{r}) dt^2 - \frac{1}{1 - 2m/r} dr^2 - r^2 d\Omega^2$$

but where the potentials $\phi_i = -m_i/r_i$ are assumed to be the fundamental quantities, as described in the introduction. The first question that arises is, how do we interpret as a potential the constant unity in $1 - 2m/r$? The answer is obvious and has been suggested by numerous researchers: Unity arises from the potential $\phi_U = -M_U/R$ due to the mass of the universe as a whole, where $M_U/R \sim 1$ (assuming G=c=1). We can thus write

$$g_{00} = \frac{M_U}{R} - \frac{2m}{r} = -\phi_U + 2\phi_{local}$$

However, this poses an obvious problem, in that the cosmic potential ϕ_U enters into g_{00} in a manner different from the local potential ϕ_{local}. Specifically, the cosmic potential has a coefficient of -1, while the local potential has a coefficient of 2.

To formulate a new theory that rectifies this problem, we define a quantity $\sigma \equiv \phi_U + \phi_1 + \phi_2 + \ldots$, where ϕ_i are the local potentials and ϕ_U is the universal potential. For simplicity only one local potential will be used. The results can be easily extended to multiple potentials. Hence, for our purposes, $\sigma = \phi_U + \phi_1 = -1 - m/r$. The quantity σ, which is in general the sum of all relevant potentials, is postulated to be more fundamental than the metric. This is the most basic distinction between Grid Metric Gravity and General Relativity, and one of the key advantages to the former.

The GMG metric components are now defined as:

$$g_{00} \equiv \frac{1}{\sigma^2}$$

$$g_{11} \equiv -\sigma^2$$

We thus have, for one local mass m:

$$g_{00} = \frac{1}{(-1-m/r)^2} = \frac{1}{1+\frac{2m}{r}+\frac{m^2}{r^2}}$$
$$g_{11} = -(-1-m/r)^2 = -(1+\tfrac{2m}{r}+\tfrac{m^2}{r^2}) \quad (1)$$

The GMG line element becomes:

$$ds^2 = \frac{1}{1+\frac{2m}{r}+\frac{m^2}{r^2}}dt^2 - (1+\tfrac{2m}{r}+\tfrac{m^2}{r^2})dr^2 - r^2 d\Omega^2 \quad (2)$$

An interesting feature of Eq (2) is that it predicts a simple form for gravitational time dilation. Specifically if $dr = d\Omega = 0$, Eq (2) gives

$$ds = d\tau = \frac{1}{|\sigma|}dt = \frac{1}{M_U/R + m/r}dt$$

indicating that time dilation is proportional to the inverse of the sum of the relevant potentials, including the cosmic potential. Similarly length contraction is proportional to the sum of all relevant potentials.

Expanding g_{00} and g_{11} in power series, we obtain the line element

$$ds^2 = [1 - 2m/r + O(m^2/r^2)]dt^2 - \frac{1}{1-2m/r+O(m^2/r^2)}dr^2 - r^2 d\Omega^2 \quad (3)$$

where $O(m^2/r^2)$ means order m^2/r^2 and above. Eq (3) is clearly identical to the Schwarzschild line element up to order m^2/r^2. The GMG line element of Eq (2) will thus reproduce, to that order, all the observationally verified results of GR. In the case of the solar system, for example, where M is the mass of the sun, $2GM/c^2 \approx 3x10^5$ cm is the sun's Schwarzschild radius, and r is on the order of millions of kilometers or 10^{11} cm, it is clear that M/r is of the order 10^{-6} and hence $M^2/r^2 \sim 10^{-12}$. Solar system tests of GR typically have errors much larger than this [34]. Based on astronomical observations, GMG may therefore be considered a viable theory of gravity.

Note that the line element of Eq (2) does not predict a black hole event horizon, since the metric is singular only at r=0. In this regard, Grid Metric Gravity resembles Newtonian gravity and classical electromagnetism.

III. THE GMG PARTICLE LAGRANGIAN

The GMG metric of Eq (2) serves to determine the particle Lagrangian $L = m'\,ds/dt$, just as it does in General Relativity. One difference is that additional local source masses can now be added in. For example, given two local masses m_1 and m_2, whose center of mass lies at the origin, we have

$$\sigma = \phi_U + \phi_1 + \phi_2 = -\frac{M_U}{R} - \frac{m_1}{r-r_1} - \frac{m_2}{r-r_2}$$

where r_1 and r_2 are the positions of the masses. From this, a metric can be constructed using

$g_{00} = 1/\sigma^2$ and $g11 = -\sigma^2$ as before. We assume for simplicity that the component $g_{22} = -r^2$ remains unchanged. This last metric component is of course accurate only for Ω equal to a constant, ie when the test particle at r is in line with the source masses, or in distant regions where $r >> r_1, r_2$. It is a sufficient approximation however for our simple presentation here. (A full spatially 3D formalism might be facilitated by rectangular coordinates. We have used spherical coordinates for easy comparison with GR.)

Applying the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} = 0$$

to the Lagrangian $L = m' ds/dt$, where ds is defined by Eq (2), gives the GMG equations of motion. These correspond to those of the Schwarzschild metric up to order m^2/r^2.

IV. GRID METRIC GRAVITY AND THE EQUATION OF STATE

Like the Schwarzschild solution of EFE for the vacuum, Grid Metric Gravity does not require an explicit Equation of State. Space-time curvature is entirely determined by gravitational mass m, which may of course include energy as well as matter. This takes the guesswork out of the results. Indeed, a seldom-noted undesirable aspect of GR is that a specific nonzero mass density distribution $\rho(r)$, when substituted into the general static solution of EFE, determines g_{11} but not g_{00}. This can be clearly seen by examining the equations for the general EFE static solution, which are:

$$\kappa\rho(r) = \frac{-(1+1/g_{11})}{r^2} + \frac{g_{11}'}{g_{11}^2 r} \qquad (4a)$$

$$\kappa p(r) = \frac{-(1+1/g_{11})}{r^2} - \frac{g_{00}'}{g_{00}g_{11}r} \qquad (4b)$$

where p(r) is the pressure. Hence, properties that depend on g_{00}, such as redshift and time dilation, are not uniquely determined by the mass distribution $\rho(r)$. An Equation of State $\rho(r) = w(r)p(r)$ must be chosen, in a manner often ad hoc, to fix these quantities. In this regard, EFE seem incomplete, as compared to the Schwarzschild metric or to Newtonian gravity, where a knowledge of ρ or mass is sufficient.

As an aside, it can be argued that the Schwarzschild solution, while not explicitly requiring an EoS, implicitly assumes w=1, or $\rho = p$. This can be shown in two ways. First, if we solve Eq (4) using w=1, this yields result $g_{00} = -1/g_{11}$, a property of the Schwarzschild metric. Second, if we solve Eq (4a) for ρ a constant, *ie*. for a universe of constant background density, we obtain the g_{11} component of what is called the *Schwarzschild-de Sitter solution* [35][28]:

$$g_{11} = \frac{-1}{1 - 2m/r - \kappa\rho r^2/3} \qquad (5)$$

where ρ is assumed in the literature to be proportional to the Cosmological Constant Λ. Since we have not yet selected an Equation of State, g_{00} remains undetermined and could in principle take any form. Now, in the case of vanishing background density, we should expect, based on physics, to obtain the Schwarzschild metric. However the Schwarzschild form is obtained only if we set w=1. Thus, it could be argued that $\rho = p$ is an implicitly assumed EoS for the Schwarzschild metric.

V. CONCLUSION

If future observational tests were to confirm GMG is more accurate than GR, this finding would have the following implications. First, it would mean Newtonian gravity employs the correct potential $\phi = -m/r$. All it lacks is the correct relativistic Lagrangian $L = m' ds/dt$. Naturally, classical physicists could not have known about this Lagrangian, since space-time had not yet been discovered. And of course the classical Lagrangian $L = T - V$ of Newtonian mechanics does not give the proper relativistic corrections, for example, to the precession of Mercury's perihelion. Nor does it predict the Shapiro time delay, gravitational time dilation, redshift, or the correct angle for the bending of light. It is the relativistic Lagrangian that predicts these phenomena. More importantly, in Grid Metric Gravity, relativistic corrections spring not from Einstein's Field Equations, but from the *Lagrangian*. This means it is conceivable that relativistic gravity could have been developed with a knowledge only of the Newtonian potential, along with a recognition that the relativistic Lagrangian could be extrapolated, using the sum of the potentials, from the Minkowski line element. No understanding of General Relativity was required.

Secondly, the field equation of GMG involves derivatives of the classical potential, not of the metric. Hence it is identical to the field equation of Newtonian gravity, which is Poisson's Equation $\nabla^2\phi = 4\pi\rho$.

A third property, discernable from the GMG metric by recalling that $1 \sim M_U/R$, is that in the limit as cosmic mass M_U approaches zero, time flows at infinite speed and space contracts to a point. That M_U is nonzero in our universe accounts for the fact that space is extended and time flows at finite speed. Thus, M_U may be interpreted as the source of Minkowski space-time. This is reminiscent of Mach's principle. Alternatively, if M_U and R approach zero simultaneously, the rate of time flow and extent of space are indeterminate. In either case, the existence of space-time depends on the cosmic mass distribution.

Fourthly, GMG is profoundly simple in that it is linear. Were GMG proven correct, the cumbersome tensor formalism of GR could be largely discarded. In its present form, however, GMG is a static model. All sources of gravity are assumed stationary, with only the test masses mobile, as is the case for the Schwarzschild metric. A dynamic GMG formalism, taking into account the speed of gravity and a covariant field equation, is still to be developed.

It is not yet known how GMG theory might affect cosmology or the galactic rotation curve. It can be said in advance that GMG would surely necessitate a replacement for the Friedman-Robertson-Walker metric of standard cosmology, which is derived from EFE in conjunction with an Equation of State. However, the static GMG theory would probably not explain the galactic rotation curve, which deviates from Newtonian predictions at large rather than small r, although a dynamic theory might do so. It also seems possible that GMG could offer methods of quantization, as an analysis of the metric suggests that matter creates space in what might be parcels of h. These are topics for future papers.

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[1] Bao-Fei Li, Madhurima Bhattacharjee, and Anzhong Wang, Phys. Rev. D 96, 084006 (2017)

[2] Arthur George Suvorov and Andrew Melatos, Phys. Rev. D 96, 064032 (2017)

[3] Gianluca Calcagni, Phys. Rev. D 96, 046001 (2017)

[4] Lee Smolin, Phys. Rev. D 96, 083523 (2017)

[5] Mordehai Milgrom, Phys. Rev. D 89, 024016 (2014)

[6] R. Monjo, Phys. Rev. D 96, 103505 (2017)

[7] Stefano Anselmi, Saurabh Kumar, Diana López Nacir, and Glenn D. Starkman, Phys. Rev. D 96, 084001 (2017)

[8] Marco Raveri, Philip Bull, Alessandra Silvestri, and Levon Pogosian, Phys. Rev. D 96, 083509 (2017)

[9] F. S. Gama, J. R. Nascimento, A. Yu. Petrov, P. J. Porfírio, and A. F. Santos, Phys. Rev. D 96, 064020 (2017)

[10] Tomonori Totani, Phys. Rev. D 96, 084062 (2017)

[11] S. A. Paston, Phys. Rev. D 96, 084059 (2017)

[12] Yun Soo Myung, Phys. Rev. D. 96, 064026 (2017)
[13] Francesco Torsello, Mikica Kocic, and Edvard Mörtsell, Phys. Rev. D 96, 064003 (2017)
[14] Fateme Rajabi and Kourosh Nozari, Phys. Rev. D 96, 084061 (2017)
[15] S. Capozziello and M. de Laurentis, Phys. Rep. 509, 167 (2011)
[16] P. H. R. S. Moraes1, and P. K. Sahoo, Phys. Rev. D 96, 044038 (2017)
[17] Georgios Kofinas and Nelson A. Lima, Phys Rev D 96, 084016 (2017)
[18] David Anderson and Nicolás Yunes, Phys. Rev. D 96, 064037 (2017)
[19] K. D. Kokkotas, R. A. Konoplya, and A. Zhidenko, Phys. Rev. D. 96, 064004 (2017)
[20] Olivier J. Veraguth and Charles H.-T. Wang, Phys Rev D 96, 084011 (2017)
[21] Juan Cayuso, Néstor Ortiz, and Luis Lehner, Phys. Rev. D 96, 084043 (2017)
[22] Luca Amendola, Nicolò Burzillà, and Henrik Nersisyan, Phys Rev. D 96, 084031 (2017)
[23] Stoytcho S. Yazadjiev, Daniela D. Doneva, and Kostas D. Kokkotas, Phys. Rev. D 96, 064002 (2017)
[24] Valerio Faraoni and Shawn D. Belknap-Keet, Phys. Rev. D 96, 044040 (2017)
[25] Manuel Hohmann, Laur Järv, and Ulbossyn Ualikhanova, Phys. Rev. D 96, 043508 (2017)
[26] Antonio De Felice, Noemi Frusciante, and Georgios Papadomanolakis, Phys. Rev. D 96, 024060 (2017)
[27] Hermano Velten and Raquel Fazolo, Phys. Rev. D 96, 083502 (2017)
[28] Valerio Faraoni, Phys. Rev. D 81, 044002 (2010)
[29] Chul-Moon Yoo and Hirotada Okawa, Phys. Rev. D 89, 123502 (2014)
[30] Eloisa Bentivegna, arXiv:1307.7673v1 [gr-qc] 29 Jul 2013.
[31] Tharake Wijenayake, Weikang Lin, and Mustapha Ishak, Phys. Rev. D 94, 083501 (2016)
[32] Scott Funkhouser, arXiv Phys 0502049 https://arxiv.org/ftp/physics/papers/0502/0502049.pdf
[33] Kathleen A. Rosser, viXra:1611.0298 (2016) http://vixra.org/abs/1611.0298
[34] Ignazio Ciufolini and John Archibald Wheeler, *Gravitation and Inertia*, Princeton University Press, 1995, p89
[35] Roman Plyatsko, Mykola Fenyk, and Volodymyr Panat, Phys. Rev. D 96, 064038 (2017)

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